Void Fractions in Two-Phase Flow

H. S. ISBIN, H. A. RODRIGUEZ, H. C. LARSON, and B. D. PATTIE

University of Minnesota, Minneapolis, Minnesota

Void fractions (fraction of the flow cross-sectional area occupied by the gas phase) have been measured for steam-water flows in an adiabatic, horizontal test section of 0.484 in. I.D. at 400, 600, 800, and 1,000 lb./sq. in. gauge. A comprehensive survey of void data for two-phase concurrent flow is included in the paper, and the data, including the Martinelli and homogeneous flow model predictions, are compared. System characteristics, involving one- and two-component flows in horizontal and vertical test sections with and without heat transfer over a range of flow ratios, total flow rates, and pressure, are too complex, and the data available are neither extensive nor precise enough to warrant the generation of over-all correlations. Use of the void data in correlating two-phase frictional pressure drops is discussed. A model has been presented for the prediction of critical flows based upon the void data, and calculations have been made for steam-water critical flows over a range of critical pressures from 15 to 2,000 lb./sq. in. abs.

A significant property of two-phase gas-liquid flow is the fraction of the cross-sectional flow area that is occupied by the gas. This fraction is known as the *void*, or *gas, fraction*. Methods of measurement have been reviewed (*7, 18, 23,* and *26*), and additional methods noted (*10, 20,* and *25*). It is recognized that such measurements yield averaged values and that very little can be said about the detailed distribution of the liquid phase in the flow channel.

Void fractions are used to determine the mixture flow density. The density term is required for the evaluation of the hydrostatic head term and is a required parameter in determining reactivity in water-cooled reactors. Use is made of the void fraction to calculate the ratio of the gas and liquid velocities. Again, in the use of this ratio, called the *slip ratio*, it is recognized that the mean velocity, calculated for each phase based upon the known volumetric flow rate and the flow areas available for each phase, is not necessarily the correct velocity to be used in the momentum and kinetic energy terms. Nevertheless the slip ratio is a very useful parameter for the evaluation of specific designs (*11, 22*).

This paper is concerned with the presentation of void-fraction measurements for adiabatic, horizontal flow of steam-water mixtures at 400, 600, 800, and 1,000 lb./sq. in. gauge and comparisons with other measurements and correlations. Further, a note is included regarding the use of the void fractions to predict critical steam-water flows.

EXPERIMENTAL

The void-fraction equipment used at the University of Minnesota has been described (*18*). The description of the high-pressure steam-water flow test stand is given in reference *16*. The measurements reported in this paper were made with a collimated beam of gamma rays (approximately 1/32 in. in diameter) produced from a 20-curie (November, 1956) thulium-170 source. A horizontal test section consisting of a ¾-in. seamless mechanical tubing with an inside diameter of 0.484 in. was used for the steam-water flows. The two-phase flows were generated by the mixing of separate sources of high-pressure steam and water flows. The test section was 8 ft. long, and the void measurements were made at an L/D ratio of about 110. Gamma ray traverses were made across the tube starting from the top to the bottom and from the bottom to the top. The readings were taken at 1/16-in. intervals except near the extreme top and bottom positions where 1/32-in. intervals were used.

The length of water in any chord position was obtained from

$$l_w = \frac{1}{\beta_w - \beta_s} \ln \frac{I_s}{I_{TP}} \qquad (1)$$

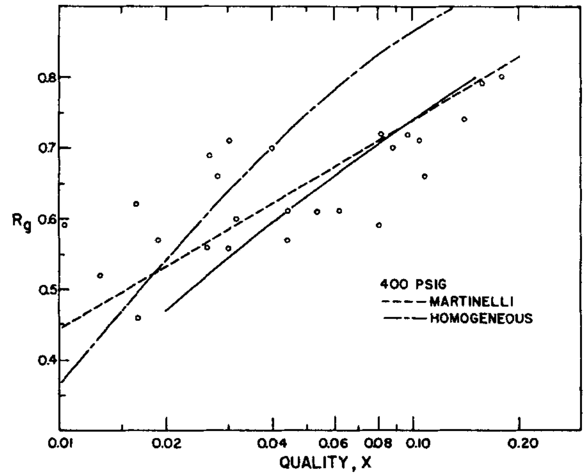

Fig. 1. Void fractions in steam-water flows at 400 lb./sq. in. gauge.

H. A. Rodriguez is with Loza Fina S. A., Guadalajara, Jalisco, Mexico, and H. C. Larson and B. D. Pattie are with Union Carbide Chemicals Company, South Charleston, West Virginia.

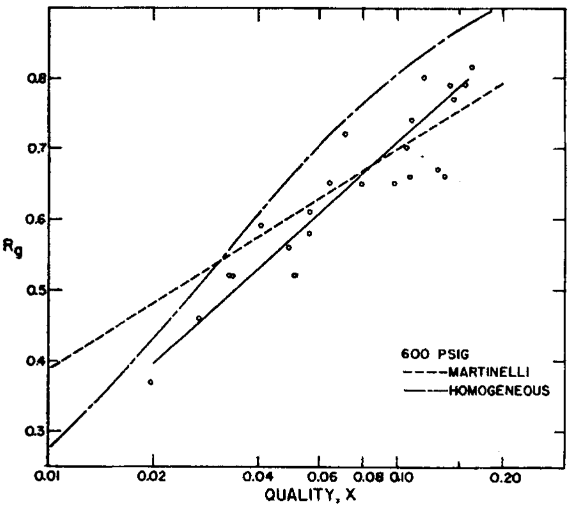

Fig. 2. Void fractions in steam-water flows at 600 lb./sq. in. gauge.

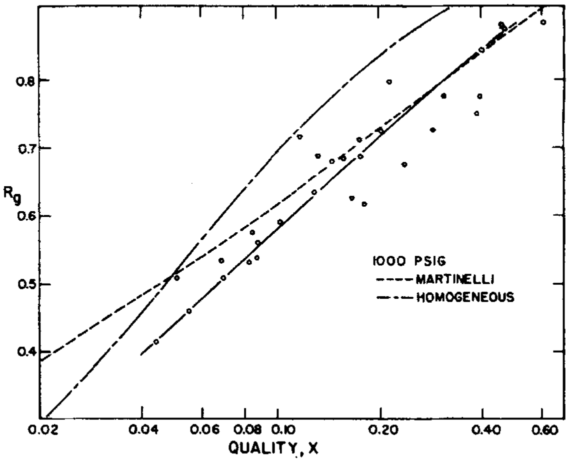

Fig. 4. Void fractions in steam-water flows at 1,000 lb./sq. in. gauge.

The absorption coefficient $(\beta_w - \beta_s)$ was experimentally determined by measuring the counting rates for the pipe filled with liquid and the pipe filled with steam. For the experimental conditions used and for a range of water temperatures from 392° to 567°F

$$(\beta_w - \beta_s) = 0.00557 \rho_w \text{ in.}^{-1} \tag{2}$$

The chordal lengths were integrated graphically over the pipe traverse to obtain the pipe cross-sectional area occupied by the liquid phase. Illustrations of the method are given in reference *18*.

Measurements were made at 400 lb./sq. in. gauge for a quality (x) range of 2 to 17.8%, total flow (W_m) range of 2,400 to 4,200 lb./hr.; 600 lb./sq. in. gauge, x from about 2 to 16.5%, W_m from 2,000 to 3,910; 800 lb./sq. in. gauge, x from 2 to 17%, W_m from 2,170 to 3,450; and at 1,000 lb./sq. in. gauge, x from 4 to 60%, and W_m from 1,330 to 2,540.

RESULTS

The experimental values for the void fractions are given in Figures 1, 2, 3, and 4, covering the measurements made at 400, 600, 800, and 1000 lb./sq. in. gauge. Included on the figures are the void fractions that would be obtained from the Martinelli curves (*24*, *15*) and from the homogeneous model (*15*). In the homogeneous model

$$R_g = \frac{x v_g}{x v_g + (1 - x) v_l}$$

where x is the quality. The extrapolation of the Martinelli curves leads to the crossing of the homogeneous curves, and modified curves have been suggested (*29*) to keep the Martinelli curves below the homogeneous curves.

The 1,000, 800, and 600 lb./sq. in. gauge curves, as shown in Figures 2, 3, and 4, were used to estimate a curve for the 400 lb./sq. in. gauge data. Further, the curve, as represented in Figure 1, was not permitted to cross the homogeneous curve. The scatter of the experimental data is believed to be mainly due to errors in maintaining constant quality during a run and in the precision of determining the low values of quality from the heat balances. It was not possible to distinguish an effect of total flow rate upon the void fraction.

The slip ratios, calculated as

$$\frac{u_g}{u_l} = \left(\frac{x}{1 - x}\right)\left(\frac{v_g}{v_l}\right)\left(\frac{1 - R_g}{R_g}\right) \tag{4}$$

are presented in Figure 5 and are based upon the solid curves of Figures 1 to 4. The experimental points given in Figures 1 to 4 lying above the solid curves have lower slip ratios, approach 1 as the points approach the homogeneous curve, and fall below 1 for points lying above the homogeneous curve.

COMPARISONS OF VOID-FRACTION DATA AND CORRELATIONS

The curves representing the University of Minnesota void data (Rodriguez and Larson's) and Popper's summary of the Argonne National Laboratory data are presented in Figure 6. Again the Martinelli curves are shown and in every case are above the Minnesota curves at the low qualities and cross them at the higher qualities. The Argonne curves are below the Minnesota curves. The Argonne data were determined for upward flow in rectangular channels with boiling heat transfer. The 1,200 lb./sq. in.

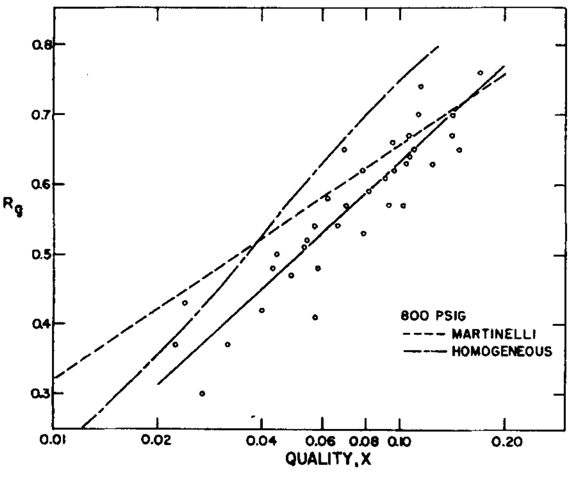

Fig. 3. Void fractions in steam-water flows at 800 lb./sq. in. gauge.

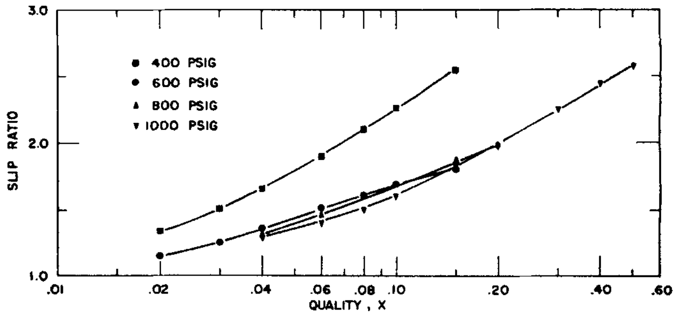

Fig. 5. Slip ratios at 400, 600, 800, and 1,000 lb./sq. in. gauge.

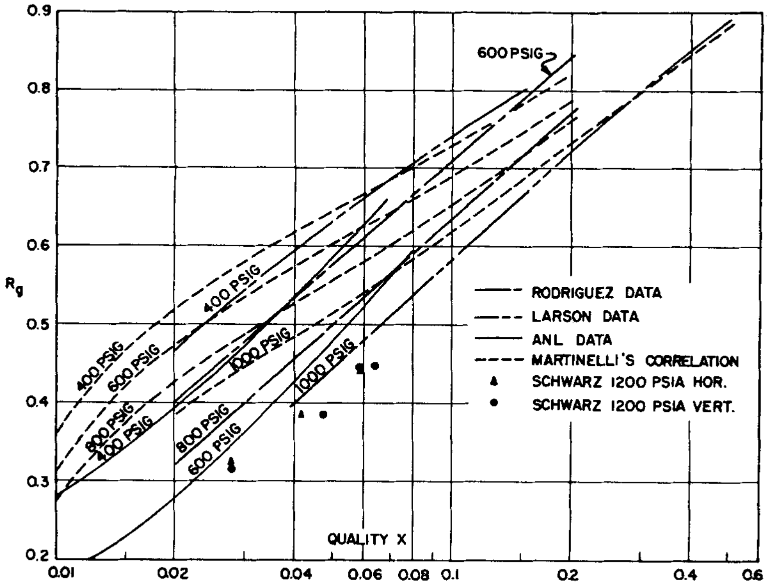

Fig. 6. Comparison of void-fraction measurements and correlations at high pressures.

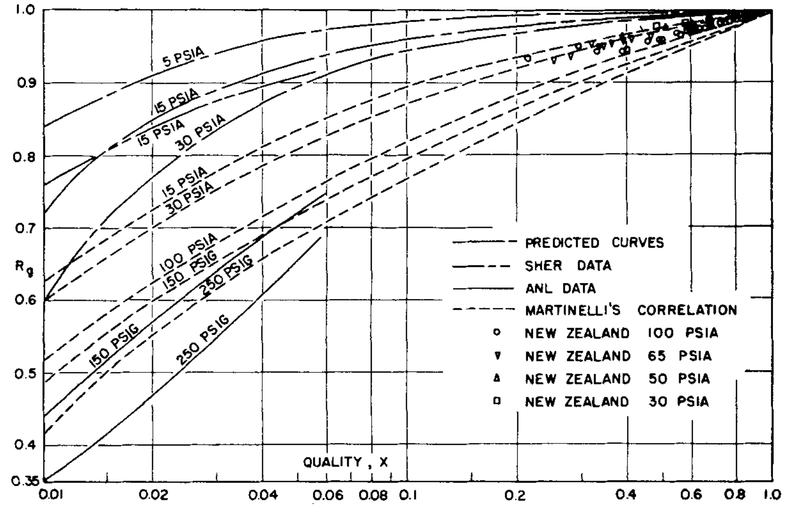

Fig. 7. Comparison of void-fraction measurements and correlations at low pressure.

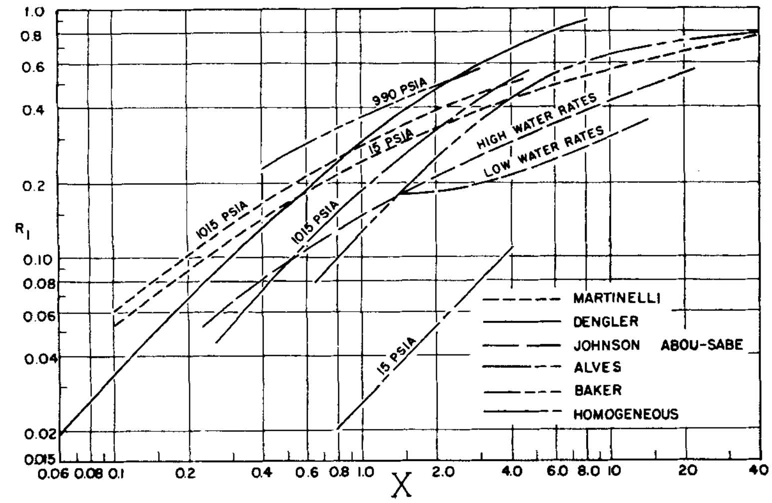

Fig. 8. R_l vs. X curves.

abs. horizontal and vertical flow data of Schwarz (*28*) are consistent with the Minnesota data.

Figure 7 presents the low-pressure void-fraction data. The New Zealand void-fraction data (*30*), determined in 4- and 6-in. horizontal pipes at 30 to 100 lb./sq. in. abs. range from 0.93 to 1 and appear to be in close agreement with the Martinelli correlation. The Argonne data fall below the Martinelli correlation, and the Minnesota data and predictions are above the Martinelli correlation. The predictions given in Figure 7 are based upon a critical flow model. (See section on critical flow.)

Liquid-fraction correlations are compared in Figure 8 as a function of the Martinelli parameter, which is the ratio of the single-phase frictional pressure drops of the liquid to the gas flow in the entire channel. Baker's curve represents data at 990 lb./sq. in. abs. (*5*), Alves's at 100 lb./sq. in. abs. (*1*), Dengler's at 7.2 to 29 lb./sq. in. abs. (*8*), and Johnson and Abou-Sabe's at 1 to 50 lb./sq. in. abs. (*19*). Curves for the homogeneous and Martinelli correlations are shown for comparison. The characteristics of the systems used by the different investigators are summarized in Table 1. Two component systems, such as gas-water and gas-oil, were employed by Alves, Martinelli, Johnson and Abou-Sabe, and Baker. (See also Note 2.) In all other systems reported, one component systems, steam-water, were used.

Armand (*2*) developed an empirical correlation for air-water mixtures in horizontal pipes, and interestingly enough a good prediction of the Minnesota data can be made over a short range of quality. (See Figure 9.) The Armand equation, for void fractions up to 0.75, is

$$R_g = 0.833 \frac{V}{V + L} \qquad (5)$$

Yagi's correlation (*34*, *35*) for upward, vertical flow of air-water and air-oil mixtures (near atmospheric pressure) is

$$\frac{R_l}{R_g} = 700\left(\frac{u_l'}{u_g'}\right)^{0.88} w_l \mu_l^{0.3} \qquad (6)$$

The Yagi correlation is not suitable for the high-pressure steam-water data but did appear adequate for the prediction of the New Zealand data for high void fraction at 30 and 100 lb./sq. in. abs.

A simplified model has been suggested by Untermeyer (*31*) as a qualitative correlation. Specific application to the 165 lb./sq. in. abs. Argonne National Laboratory data indicated that the predicted values are much higher than the experimental ones.

The Zmola-Bailey correlation (*3*) when applied to steam-water flow is limited to low qualities at low pressures. At higher pressures the predicted values were lower than the Argonne National Laboratory data at 165 and 615 lb./sq. in. abs.,

TABLE 1. SURVEY OF VOID-FRACTION INVESTIGATIONS

Reference	X range*	Pressure, lb./sq. in. abs.	Conditions	Geometry	Method of void measurement
(*1*)	0.65–100	100	FC NHT	HP 1 in.	Isolate test section and weigh
(*24*)	0.10–50	near atm.	FC NHT	HP ½ and 1 in.	Isolate test section and weigh
(*8*)	0.05–10	7.2 to 29	FC WHT	VT 1 in.	Radioisotope in fluid mixture
(*19*)	0.2–15	1–50	FC NHT	HT 1 in.	Isolate test section and weigh
(*5*)	0.4–3 Quality	990	FC NHT	HP 8 and 10 in.	Separate phases and weigh
(*22*)	0.012–0.079	165, 265 415, 615	NC WHT	VC ½ and 1 in.	Tm^{170} γ attn. Cross section
(*32*)	0.01–0.074	165, 265 215, 615	NC WHT	VC ¼ and 2 in.	Tm^{170} γ attn. Cross section
(*23*)	0.019–0.082	115, 274 314, 414 514, 614	NC WHT	VC Four 7/16 and 3 11/16 in.	Tm^{170} γ attn. Cross section
(*7*)	0.012–0.140	614	NC WHT	VC Four 7/16 and 3 11/16 in.	Tm^{170} γ attn. Cross section
(*9*)	0–0.04	2000	FC WHT	VC 1 by 0.103 in.	Tm^{170} γ attn. Cross section
Rodriguez	0.02–0.20	415, 615 815	FC NHT	HP 0.484 in.	Tm^{170} γ attn. Chordal lengths
Larson	0.04–0.40	1,015	FC NHT	HP 0.484 in.	Tm^{170} γ attn. Chordal lengths
Eddy	0.012–0.048	25	FC NHT	HP 1.049 in.	Se^{75} γ attn. Chordal lengths
Sher	0.004–0.042	15	NC NHT	VT 0.872 in.	Se^{75} γ attn. Chordal lengths
(*30*)	0.21–1.0	100, 65 50, 30	FC NHT	HP 4 and 6 in.	Dye injection and sound
(*28*)	0.028–0.064	1,200	HC NHT	HT and VT 60 mm.	Ir^{192} γ attn. Chordal lengths

Note 1. The following symbols were used in this table: V—vertical, H—horizontal, P—pipe, T—tube, C—rectangular channels, FC—forced circulation, NC—natural circulation, WHT—with heat transfer, NHT—no heat transfer.

2. See Armand (*1*), Zmola-Bailey (*3*), Yagi (*34*, *35*), Untermeyer (*31*), Bergelin and Gazley (*4*), and Govier et al. (*12*).

*X = Martinelli's parameter.

and lower than the Minnesota data at 815 lb./sq. in. abs.

The effect of flow rate cannot be distinguished in the Minnesota data; however, Lottes and Flinn (*22*) have suggested that although the slip-ratio dependence upon power density, voids, quality, recirculation flow, pressure, and geometry is not fully understood, a useful empirical relation may be used showing the slip ratio dependence upon total mass velocity and system pressure. Marchaterre (*23*) found that in his heated test sections, with inlet fluid at saturation temperature, the assumption of constant slip along a channel length was reasonable. The initial rise of the slip ratio with length of heated section is extended if the liquid is subcooled and if the vaporizing flow is at low pressure with higher void fractions. Cook, in his studies at 614.7 lb./sq. in. abs. (*7*), found that there is an increase in the slip ratio with length of the heated channel and that the change is related to the rate of vaporization. At the initiation of vaporization the slip ratio was approximately 1.5 and appeared to be independent of inlet velocity. Steam-void fractions up to 10% were found at positions in the channel where a heat balance would indicate zero quality. The presence of steam was believed to be indicative of stratification. (See also reference *9*.) Petrick (*26*) suggests the following approximation for the slip ratio:

$$\frac{u_g}{u_l} = K u_{l0}'^{N} \qquad (7)$$

where N is an exponent that varies with pressure (about 0.31 at 150 lb./sq. in abs., 0.2 at 1,500 lb./sq. in. abs., and zero at the critical pressure for steam-water flows). The approximation was used to develop a relationship between static pressure and void fractions before and after a contraction or expansion.

An analytical model has been developed for two-phase flow by Westmoreland (*33*). The model differs from Levy's model (see reference *15* for comparison of models) in that at the liquid-gas interface it is assumed that the phase velocities and the shearing stresses are equal. Although the equations for predicting the two-phase frictional drops will not show a dependency on total flow (other than flow types), the predictions for the slip ratio, for example for steam-water at 615 lb./sq. in. abs., are good.

CRITICAL-FLOW PREDICTIONS

Measured and predicted critical flows of steam-water mixtures at low pressures are given in reference *17*. The approximations used in developing the model are noted, and in the examples given use was made of the Martinelli void correlations. At low qualities (less than 10%) the deviations between the predicted critical flows and the observed values were significant. It was found that slight changes in the void-fraction correlation could improve the correlation. For example, the difference between measured critical flows and predicted flows could be made less than 3% for the entire quality range if the void fraction curves were changed as noted

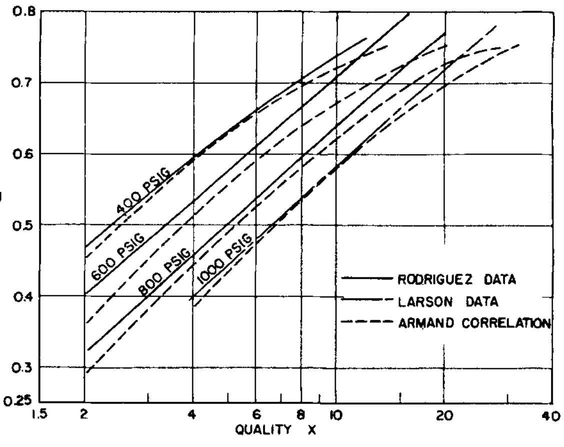

Fig. 9. Comparison of University of Minnesota void data with Armand correlation.

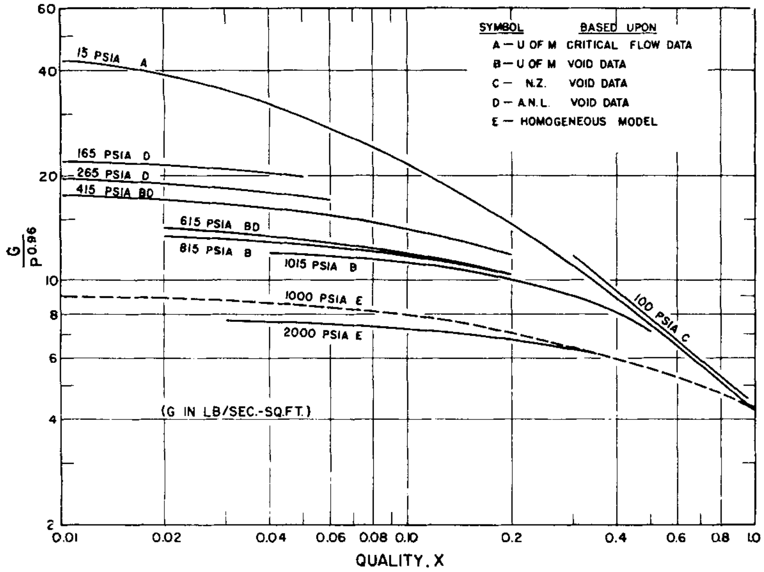

Fig. 10. Critical-flow predictions.

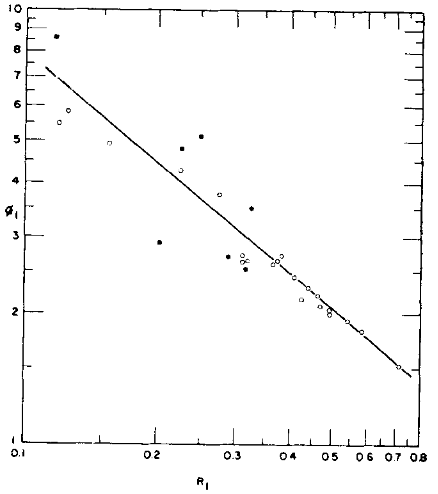

Fig. 11. ϕ_l vs. R_l for 1,000 lb./sq. in. gauge data. (Solid points lying above the curve represent the lowest flow rates and those below, the highest flow rate.)

in Figure 7, under Predicted Curves.*

Figure 10 is a presentation of predicted, critical steam-water mass-flow rates as a function of quality and with the critical pressure a parameter. The form of the plot with $G/P^{0.96}$ used was adapted from the previous correlations at low pressure (*17*). The Minnesota, Argonne, and New Zealand void data were used to predict critical flows over the range of data available. Critical flows based upon the 415 and 615 lb./sq. in. abs. Argonne and Minnesota data could be grouped into single curves at each pressure. The 2,000 lb./sq. in. abs. curve is based on the homogeneous model, which investigators at Battelle (*9*) used to correlate their void-fraction data at 2,000 lb./sq. in. abs. above a quality of 0.03. For comparison at lower pressure the 1,000 lb./sq. in. abs. curve, based upon the homogeneous model, lies about 40% below the predicted curve. Data are not yet available to check critical-flow predictions at pressures above 45 lb./sq. in abs., and thus the usefulness of the model has not been established.

PRESSURE-DROP–VOID-FRACTION RELATIONSHIPS

Several modifications of the friction-factor method have been suggested for correlating two-phase frictional pressure drops. Mosher (*36*) started with the Linning model (*21*), which noted that the frictional forces are exhibited at the wall only and that with annular flow the correct velocity to use in the Fanning equation is the mean velocity of the liquid. Liquid density and liquid viscosity are to be used as well as the pipe diameter. Some comparisons of the predicted to measured steam-water pressure drops were made, based upon the Martinelli void-fraction correlation; however, the predictions were not much better than the homogeneous model.

An independent analysis was made by Flinn and Lottes (*22*) in which they, too, assumed that the frictional effect of the two-phase flow is due primarily to the drag of the water phase along the wall. For small qualities

$$\phi_{l0}^2 = \frac{(\Delta P_{TPF}/\Delta L)}{(\Delta P_{l0}/\Delta L)} \simeq \frac{1}{(1 - R_g)^2} \quad (8)$$

The two-phase frictional pressure drop gradient is taken as

$$\frac{dP_{TPF}}{dL} = \frac{f_l \rho_l u_l^2}{2gD} \quad (9)$$

where u_l equals the superficial velocity divided by the liquid fraction, $(1 - R_g)$. If the Reynolds number defining the friction factor is based upon the total liquid flow and pipe diameter, then the friction factor for the two-phase flow model is the same as that used in calculating ΔP_{l0}. With this restriction and the inclusion of the quality

$$\phi_{l0}^2 = \left(\frac{1 - x}{1 - R_g}\right)^2 \quad (10)$$

Petrick's experiments (*26*) (air-water upward flow at atmosphere pressure) indicated that this model was not satisfactory, particularly over wide ranges of flow rate.

If it is assumed that $f_l \propto 1/u_l^m$ and if the two-phase frictional pressure drop is compared with the pressure drop due to just the liquid flow, then

$$\phi_l^2 = \left(\frac{\Delta P_{TPF}/\Delta L}{\Delta P_l/\Delta L}\right) = \left(\frac{1}{1 - R_g}\right)^{2-m} \quad (11)$$

Chisholm and Laird (*6*) found that the majority of their data (horizontal air-water flows in 1-in. bore) for smooth and galvanized tubes lie within ±20% of the equations

$$\phi_l^2 = \frac{0.8}{(1 - R_g)^{1.75}}$$

for smooth tubes (12)

and

$$\phi_l^2 = \frac{0.8}{(1 - R_g)^{1.875}}$$

for galvanized tubes (13)

Hoopes (*14*) considered the relationship

$$\phi_{l0}^2 = \frac{f_{TPF} r_{HTP}}{f_l r_{Hl}} \left(\frac{1 - x}{1 - R_g}\right)^2 \quad (14)$$

The plot of ϕ_{l0}^2 vs. $[(1 - x/(1 - R_g)]^2$ exhibits considerable scatter owing in part to the difficulty in accurately determining the frictional two-phase pressure drops and the quality.

Other references on the use of

$$\phi_l^a R_l^b = C \quad (15)$$

are given by Moen (*37*) and Sher (*18*).

Figure 11 is a plot of the ϕ_l vs. R_l Minnesota steam-water data at 1,000 lb./sq. in. gauge. A very definite flow-rate effect is noted. The solid points lying above the curve represent the lowest flow rates, and those below the curve represent the highest. The flow-rate effect is due to the influence of the total flow on the frictional two-phase pressure drop, and a limited correlation is presented in reference *16*. Petrick (*26*) also noted a flow-rate effect in his air-water data.

An empirical correlation was found by Petrick (*26*) to be satisfactory in correlating air-water data at atmospheric pressure and boiling steam water at 2,000 lb./sq. in.

*The 15 lb./sq. in. abs. curve approximates Sher's data (*18*).

$$\phi_{l0}^2 = \frac{KR_g^n}{\left(\frac{G}{10^6}\right)^{0.76}} \quad (16)$$

where G is the mass flow rate in lb./(hr.)(sq. ft.); $K = 9.6$ and $n = 1.25$ for air-water; $K = 8.5$ and $n = 1.4$ for steam-water at 2,000 lb./sq. in.; n is a function of G for range of variables beyond R_g from 0.3 to 0.7, and G from 0.3 to 2×10^6. The Larson 1,000 lb./sq. in. data were approximated by Equation (16) with n about 1.4 and K about 14. Further work is being done to check the correlation over a wider pressure range for steam-water flows.

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NOTATION

Pressure Drops and Pressure-Drop Ratios

$\Delta P_{TPF}/\Delta L$ = frictional two-phase pressure drop per unit length
$\Delta P_{l0}/\Delta L$ = frictional pressure drop per unit length calculated for the total flow as liquid flow in the entire channel
$\Delta P_l/\Delta L$ = frictional pressure drop per unit length calculated for just the liquid flow in the entire channel
$\Delta P_g/\Delta L$ = frictional pressure drop per unit length calculated for just the liquid flow in the entire channel
ϕ_{l0}^2 = $(\Delta P_{TPF}/\Delta L)/(\Delta P_{l0}/\Delta L)$, ratio of two-phase frictional pressure drop per unit length to the frictional pressure gradient calculated for the total flow as liquid
ϕ_l^2 = $(\Delta P_{TPF}/\Delta L)/(\Delta P_l/\Delta L)$
X = Martinelli parameter, $(\Delta P_l/\Delta L)/(\Delta P_g/\Delta L)$

Flow Quantities

w_g and w_l = mass flow of gas and liquid, respectively ($W_m = w_l + w_g$), kg./(sq. meter)(sec.) in Equation (6)
V and L = volumetric flow rate of gas and liquid, respectively
u_g' and u_l' = superficial velocities of the gas phase and liquid phase, respectively, meter/sec. in Equation (6)
u_g and u_l = velocity of gas phase and liquid phase, respectively, based upon cross-sectional area available for flow of phase
G = mass flow rate, lb./(hr.)(sq. ft.)
u_g/u_l = slip ratio
u_{l0} = superficial entering velocity

Others

β_w and β_s = linear gamma-absorption coefficient for water and steam, respectively
μ_l and μ_g = viscosity of liquid and gas phase, respectively, kg./(meter)(sec.) in Equation (6)
ρ_l or ρ_w and ρ_g = density of liquid and gas, respectively, lb./cu. ft.
A = cross-sectional area of pipe for flow
D = pipe diameter
f = Fanning friction factor
f_l = friction factor
g = conversion factor
K, m, n, N = constants (given in defining equations)
l_w = chordal length of water in tube
I_{TP}, I_s, I_w = radiation intensity transmitted through two-phase flow, steam flow, and water flow respectively
P = pressure at critical flow
p = pressure
r = hydraulic radius for the flows
r_{HTP}, r_{Hl} = hydraulic radius for two-phase flow and liquid flow, respectively
R_g = void fraction, fraction of cross-sectional flow area occupied by gas phase
R_l = liquid fraction = $1 - R_g$, fraction of cross-sectional flow area occupied by liquid phase
v_l and v_g = specific volume of liquid and gas phase, respectively
x = quality

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